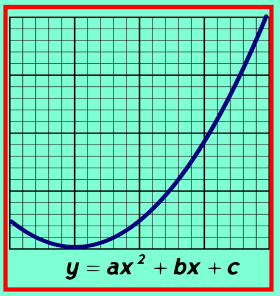
Math 125

Fall 2021

Lecture 35



Class Qz 29

1) Simplify: $(\sqrt{5} - \sqrt{2})^2$

$= (\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2}) = \sqrt{25} - \sqrt{10} - \sqrt{10} + \sqrt{4} = 5 - 2\sqrt{10} + 2 =$

$\rightarrow = \boxed{7 - 2\sqrt{10}}$ ✓

2) Solve & check: $\sqrt{2x-1} - 2 = 5$

$\sqrt{2x-1} = 5 + 2$ ✓

$\sqrt{2x-1} = 7$

$(\sqrt{2x-1})^2 = (7)^2$

$\rightarrow 2x - 1 = 49$

$2x = 50$

$\boxed{x = 25}$ ✓

$\{25\}$

Check:

$\sqrt{2(25)-1} - 2 = 5$

$\sqrt{49} - 2 = 5$

$7 - 2 = 5$

$5 = 5$ ✓

Simplify

1) $(4-3i)^2 = (4-3i)(4-3i) = 16-12i-12i+9i^2$

$= 16-24i+9(-1)$

$= 16-24i-9$

$= \boxed{7-24i}$

Re. Part 7

Im. " -24

2) $\dfrac{1-i}{3+4i} = \dfrac{(1-i)(3-4i)}{(3+4i)(3-4i)}$

$= \dfrac{3-4i-3i+4i^2}{9-12i+12i-16i^2}$

$= \dfrac{3-7i+4(-1)}{9-16(-1)} = \dfrac{3-7i-4}{9+16} = \dfrac{-1-7i}{25} = \boxed{\dfrac{-1}{25}-\dfrac{7}{25}i}$

3) $\dfrac{-10i}{8-6i} = \dfrac{-10i(8+6i)}{(8-6i)(8+6i)} = \dfrac{-80i-60i^2}{64+48i-48i-36i^2}$

$= \dfrac{-80i-60(-1)}{64-36(-1)}$

$= \dfrac{60-80i}{100}$

$= \dfrac{60}{100}-\dfrac{80}{100}i$

$= \boxed{\dfrac{3}{5}-\dfrac{4}{5}i}$

4) $i^{250} = (i^2)^{125} = (-1)^{125} = \boxed{-1}$

5) $i^{125} = i^{124}\cdot i = (i^2)^{62}\cdot i$

$= (-1)^{62}\cdot i = 1i = \boxed{i}$

Removing radicals from denominator or numerator is called rationalizing.

Rationalize the denominator:

$$\frac{4}{\sqrt{10}} = \frac{4 \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}} = \frac{4\sqrt{10}}{\sqrt{100}} = \frac{{}^{2}\cancel{4}\sqrt{10}}{\cancel{10}_{5}} = \boxed{\frac{2\sqrt{10}}{5}}$$

Rationalize the numerator

$$\frac{3\sqrt{6}}{2} = \frac{3\sqrt{6} \cdot \sqrt{6}}{2 \cdot \sqrt{6}} = \frac{3\sqrt{36}}{2\sqrt{6}} = \frac{3 \cdot \cancel{6}^{3}}{\cancel{2}\sqrt{6}} = \boxed{\frac{9}{\sqrt{6}}}$$

Rationalize the denominator

$$\frac{2\sqrt{3}}{\sqrt{3}-1} = \frac{2\sqrt{3}\ (\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{9} + 2\sqrt{3}}{\sqrt{9} + \cancel{\sqrt{3}} - \cancel{\sqrt{3}} - 1} = \frac{6+2\sqrt{3}}{3-1}$$

$$= \frac{6+2\sqrt{3}}{2} = \frac{{}^{3}\cancel{6}}{\cancel{2}_{1}} + \frac{\cancel{2}\sqrt{3}}{\cancel{2}_{1}} = \boxed{3+\sqrt{3}}$$

Rationalize the denominator

$$\frac{5}{\sqrt{7}+\sqrt{2}} = \frac{5(\sqrt{7}-\sqrt{2})}{(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})} = \frac{5(\sqrt{7}-\sqrt{2})}{\sqrt{49}-\sqrt{14}+\sqrt{14}-\sqrt{4}}$$

$$= \frac{5(\sqrt{7}-\sqrt{2})}{7-2}$$

$$= \frac{\cancel{5}(\sqrt{7}-\sqrt{2})}{\cancel{5}}$$

$$= \boxed{\sqrt{7}-\sqrt{2}}$$

Rationalize the denominator

$$\frac{2\sqrt{5}+\sqrt{2}}{2\sqrt{5}-\sqrt{2}} = \frac{(2\sqrt{5}+\sqrt{2})(2\sqrt{5}+\sqrt{2})}{(2\sqrt{5}-\sqrt{2})(2\sqrt{5}+\sqrt{2})}$$

$$= \frac{4\sqrt{25}+2\sqrt{10}+2\sqrt{10}+\sqrt{4}}{4\sqrt{25}+\cancel{2\sqrt{10}}-\cancel{2\sqrt{10}}-\sqrt{4}}$$

$$= \frac{4\cdot 5+4\sqrt{10}+2}{4\cdot 5-2} = \frac{22+4\sqrt{10}}{18}$$

$$= \frac{22}{18}+\frac{4\sqrt{10}}{18} = \boxed{\frac{11}{9}+\frac{2\sqrt{10}}{9}}$$

Solve and Check:

$\sqrt{x-3}+6=5$

$\sqrt{x-3}=5-6$

$\sqrt{x-3}=-1$

$\emptyset$

$(\sqrt{x-3})^2=(-1)^2$

$x-3=1$

$x=4$ ← Extraneous Solution

Check:

$\sqrt{4-3}+6=5$

$\sqrt{1}+6=5$

$1+6=5$

$7=5$ False

Solve & check

$x+\sqrt{26-11x}=4$

$\sqrt{26-11x}=4-x$

$(\sqrt{26-11x})^2=(4-x)^2$

$26-11x=(4-x)(4-x)$

$26-11x=16-4x-4x+x^2$

$26-11x=16-8x+x^2$

$x^2-8x+16-26+11x=0$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x+5=0$ $\quad x-2=0$

$x=-5$ $\quad x=2$

$\{-5,2\}$

Check $x=-5$ ✓

$-5+\sqrt{26-11(-5)}=4$

$-5+\sqrt{26+55}=4$

$-5+\sqrt{81}=4$

$-5+9=4$

$4=4$ ✓

Check $x=2$

$2+\sqrt{26-11(2)}=4$

$2+\sqrt{26-22}=4$

$2+\sqrt{4}=4$

$2+2=4$

$4=4$ ✓

Solve: $\sqrt{6x+7} - x = 2$

$\sqrt{6x+7} = x+2$

$(\sqrt{6x+7})^2 = (x+2)^2$

$6x+7 = (x+2)(x+2)$

$6x+7 = x^2+2x+2x+4$

$6x+7 = x^2+4x+4$

$x^2+4x+4-6x-7=0$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$x+1=0$ $\quad x-3=0$

$\checkmark x=-1$ $\quad \checkmark x=3$

$\{-1, 3\}$

Check $x=-1$

$\sqrt{6(-1)+7} - (-1) = 2$

$\sqrt{-6+7} + 1 = 2$

$\sqrt{1} + 1 = 2$

$1+1=2\checkmark$

Check $x=3$

$\sqrt{6(3)+7} - 3 = 2$

$\sqrt{18+7} - 3 = 2$

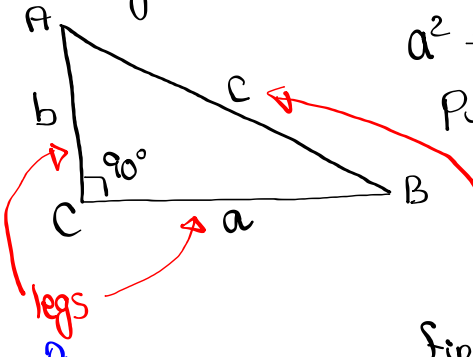
$\sqrt{25} - 3 = 2$

$5-3=2\checkmark$

Working with right-Triangle:

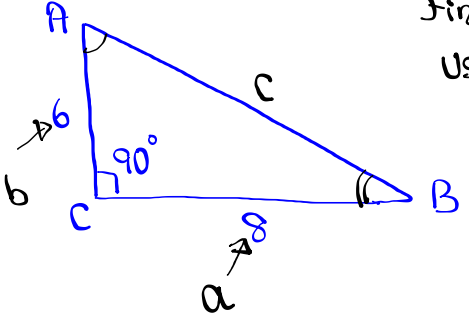
$a^2 + b^2 = c^2$

Pythagorean Theorem.



Hypotenuse

legs



Find C:

Using Pythagorean thm

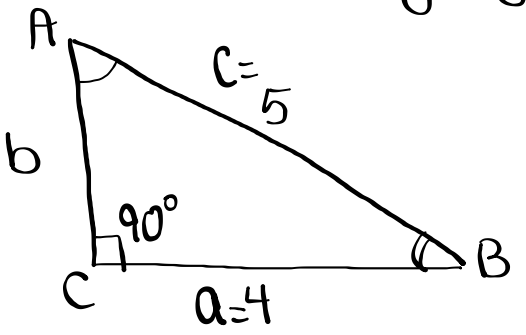
$a^2 + b^2 = c^2$

$6^2 + 8^2 = c^2$

$100 = c^2$

$c = 10$

Find the missing leg:



Find b

Pythagorean thrm

$a^2+b^2=c^2$

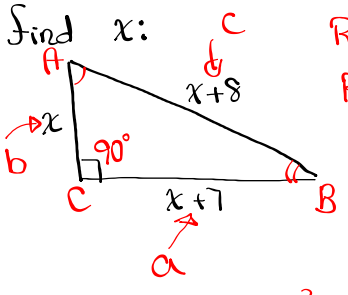
$4^2+b^2=5^2$

$16+b^2=25$

$b^2=25-16$

$b^2=9$ $\boxed{b=3}$

Find x:



Right-Triangle Pythagorean Thrm

$a^2+b^2=c^2$

$(x+7)^2+x^2=(x+8)^2$

$(x+7)(x+7)+x^2=(x+8)(x+8)$

$x^2+7x+7x+49+x^2=x^2+8x+8x+64$

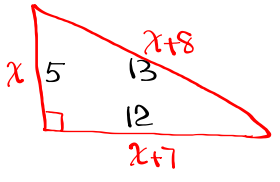
$2x^2+14x+49-x^2=16x-64=0$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x-5=0$ $\boxed{x=5}$

$x+3=0$ $x=-3$



Check

$5^2+12^2=13^2$ ✓

Class QZ 30

1) Simplify: $3(2-4i) - 2(4+3i)$

$= 6 - 12i - 8 - 6i = \boxed{-2 - 18i}$

2) Simplify: $(3-5i)^2 = (3-5i)(3-5i)$

$= 9 - 15i - 15i + 25i^2$

$= 9 - 30i + 25(-1)$

$= \boxed{-16 - 30i}$

3) Solve $x + \sqrt{x+1} = 1$

$\sqrt{x+1} = 1 - x$

$(\sqrt{x+1})^2 = (1-x)^2$

$x + 1 = (1-x)(1-x)$

$x + 1 = 1 - x - x + x^2$

$x^2 - 2x + 1 - x - 1 = 0$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\boxed{x=0}$ ✓

$x - 3 = 0$

$x = 3$ ✗

$\{0\}$